Homework problems # 2

1. Construct the Noether current in scalar electrodynamics described by the \( \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* (D^\mu \varphi) - m^2 \varphi^* \varphi \) with \( D_\mu \varphi \equiv (\partial_\mu - i e A_\mu) \varphi \). Show that the current is conserved for fields which satisfy appropriate equations of motion.

2. Find the Noether current corresponding to space-time translations and the symmetric energy-momentum tensor in scalar electrodynamics.

3. Derive equations of motion for the Yang-Mills theory with the gauge group SU(2) interacting with SU(2) doublet of scalar fields.

4. For the same theory as above, find the symmetric energy-momentum tensor and show that it is conserved if equations of motion are satisfied.