STW 100 lat potem

Czyli NIE o elektrodynamice, a TAK o ciałach w ruchu

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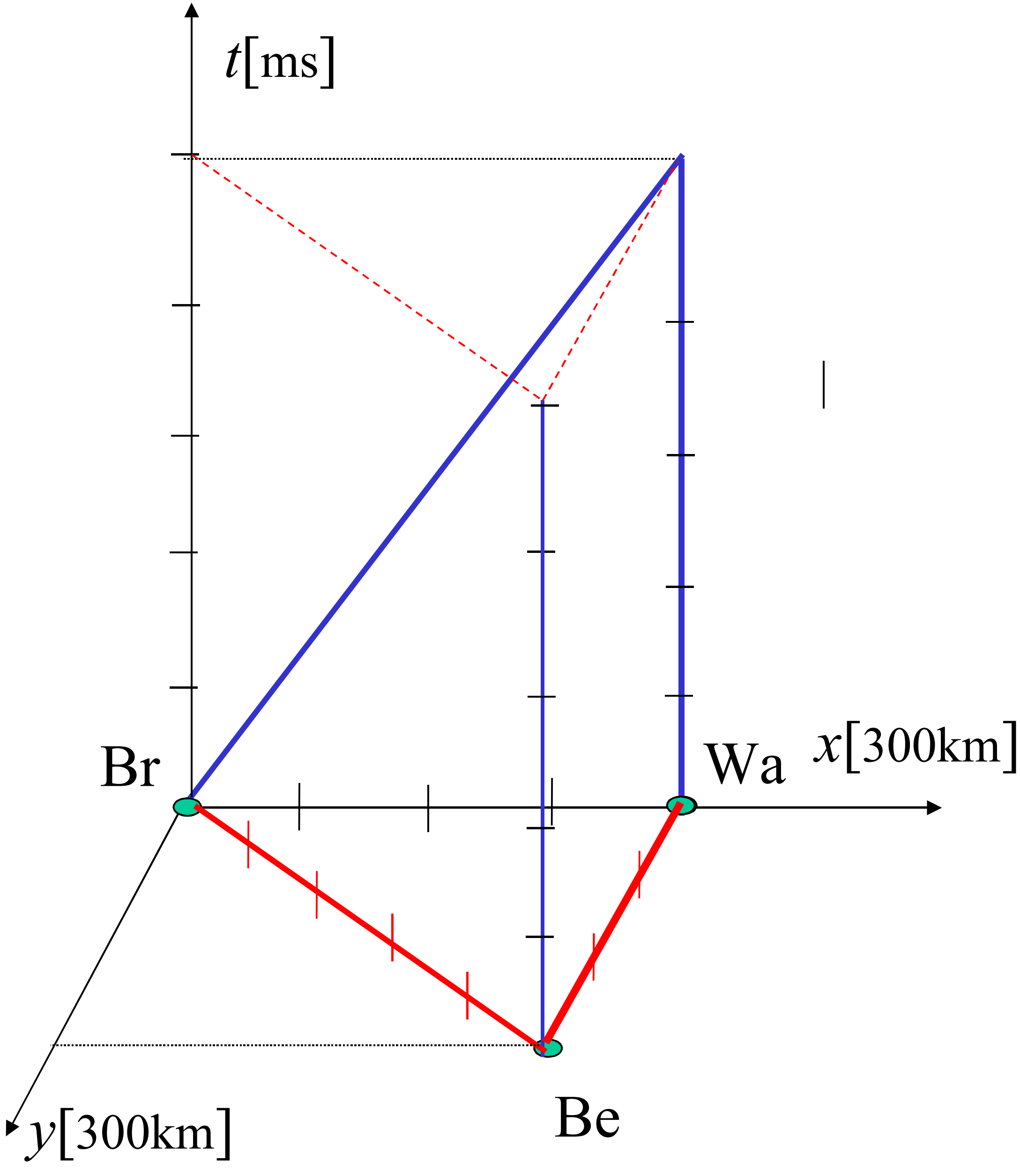
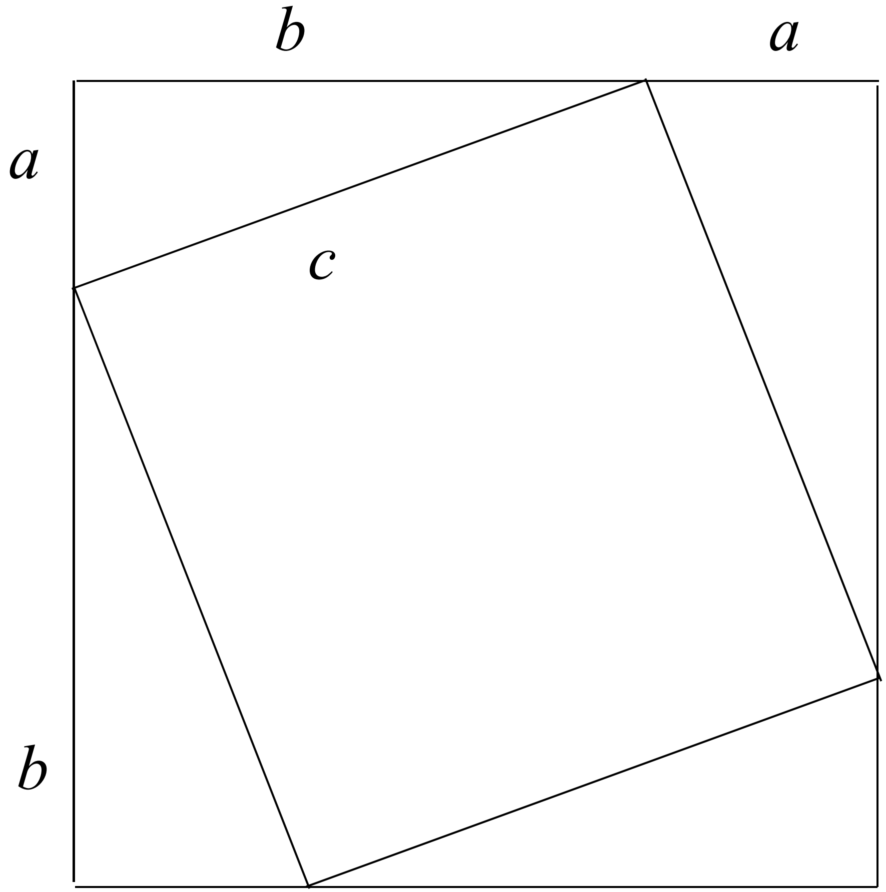
Europa



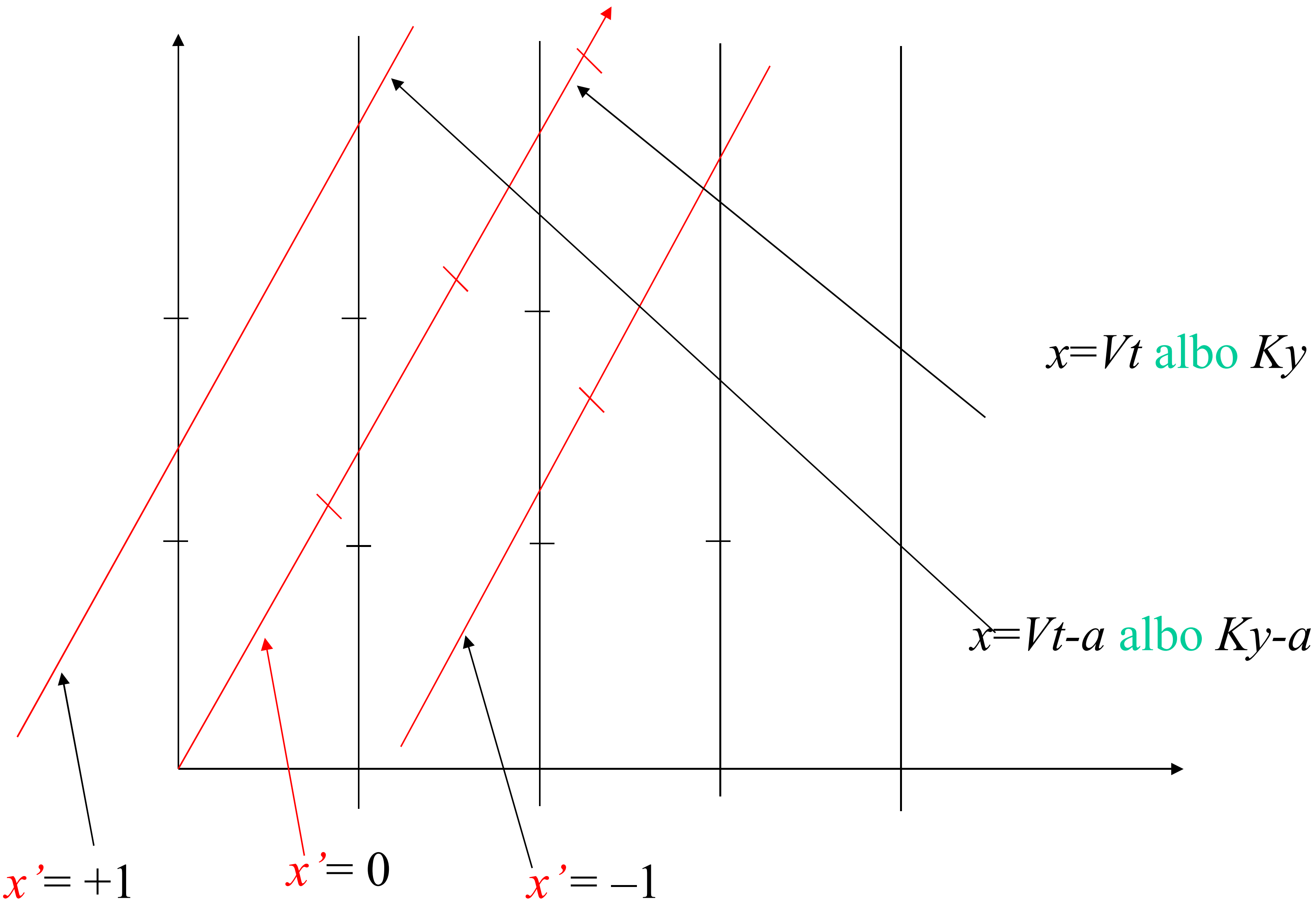
Pitagoras

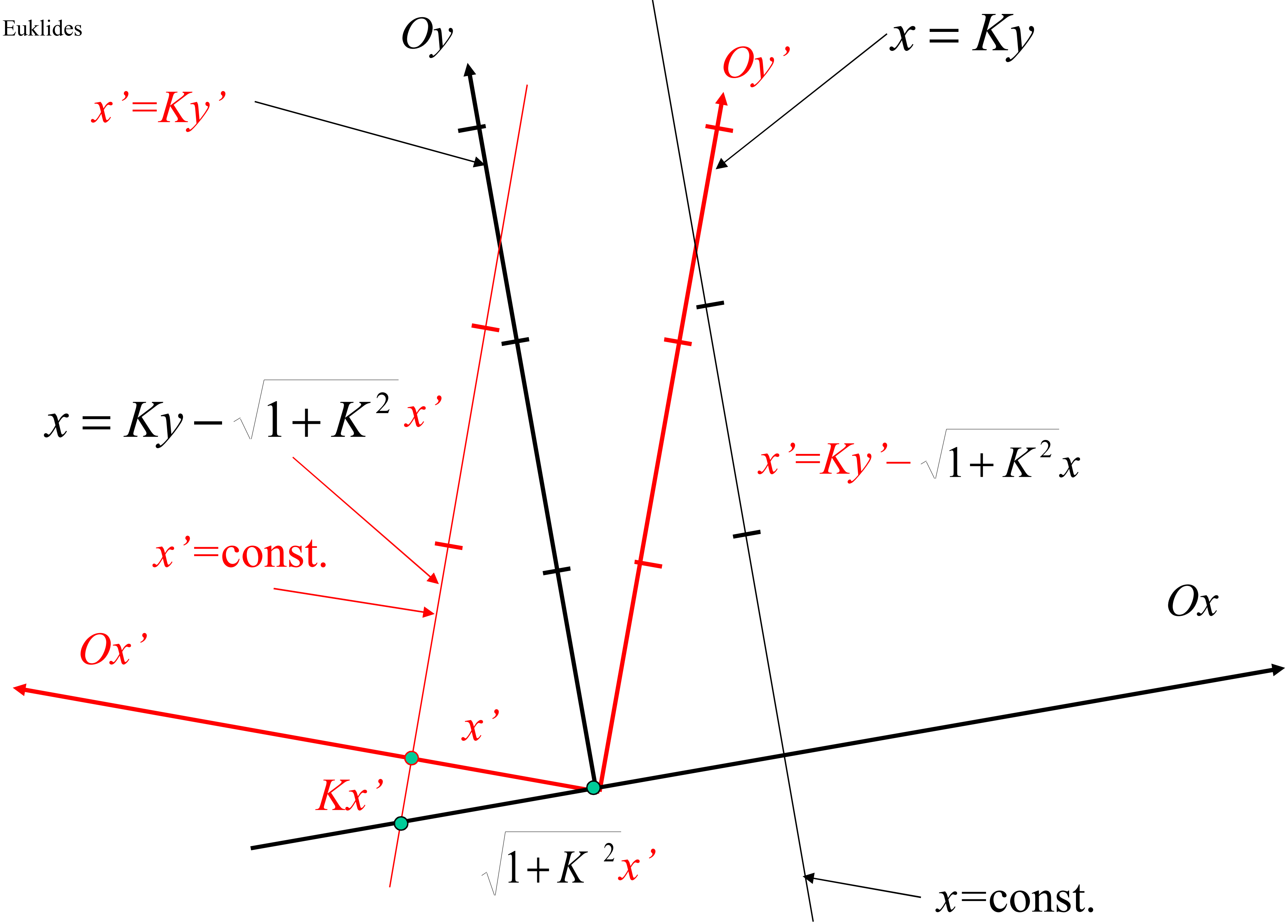


$$(a+b)^2 = c^2 + 4\cdot\frac{1}{2}ab$$

$$a^2 + b^2 + 2ab = c^2 + 4\cdot\frac{1}{2}ab$$

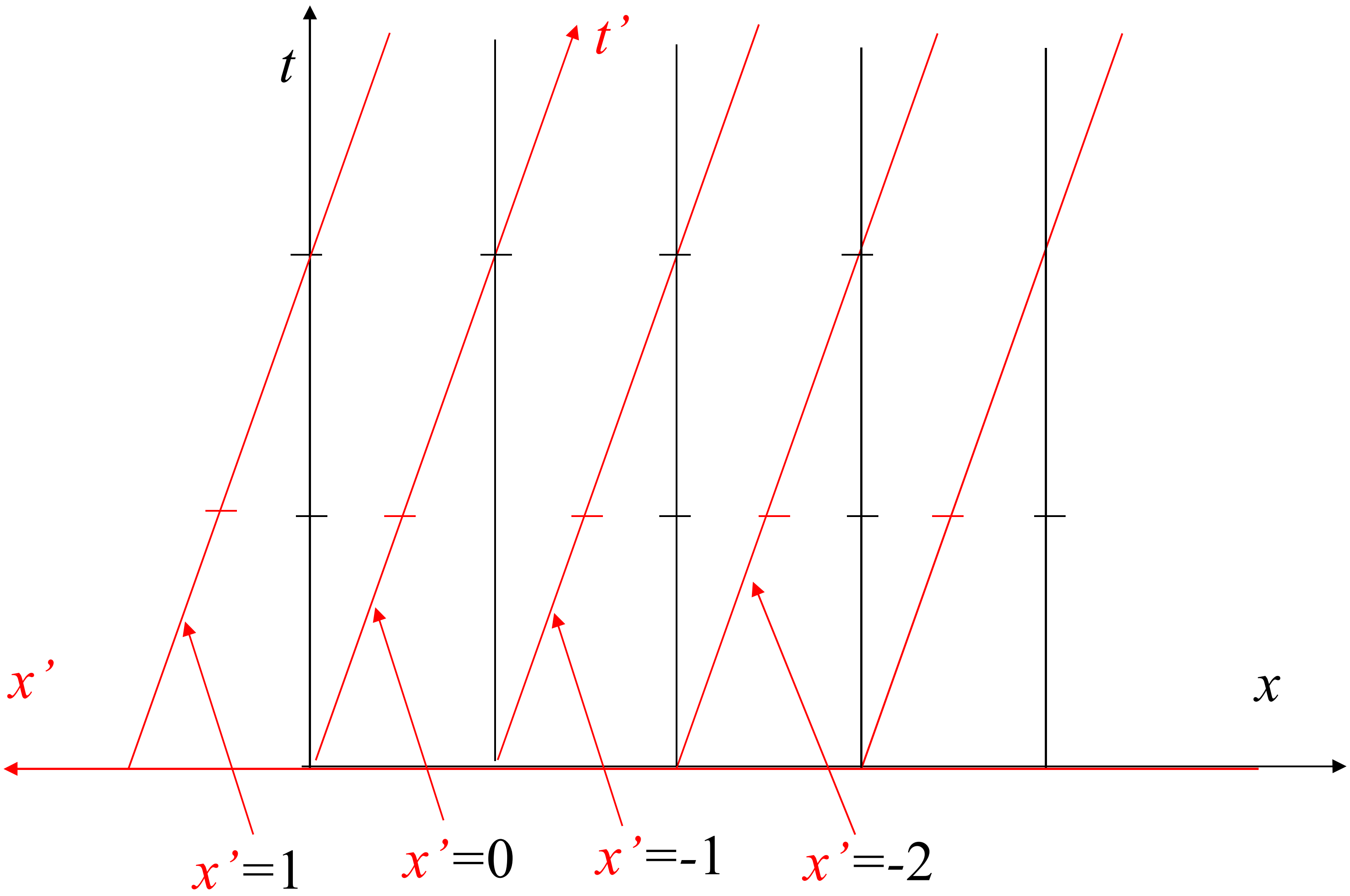
Stałe *x'*





Oy
x = Ky
Oy'
x'=Ky'
x = Ky − √(1+K²) x'
x'=Ky'− √(1+K²) x
x'=const.
Ox
Ox'
x'
Kx'
√(1+K²) x'
x=const.

Świat Galileusza



$t = t'$ $\quad$ $x = V\,t - x'$ $\quad$ $x' = V\,t' - x$

$$x = Vt - ax' \qquad x = Ky - ax'$$

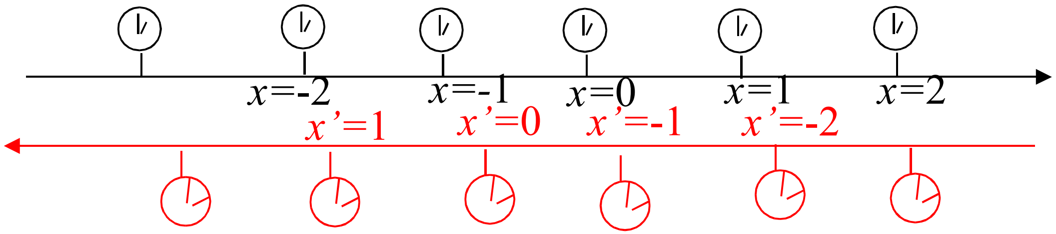
$$x' = Vt' - ax \qquad x' = Ky' - ax$$

Galileusz: $a=1$,

Euklides: $a = \sqrt{1+K^2}$

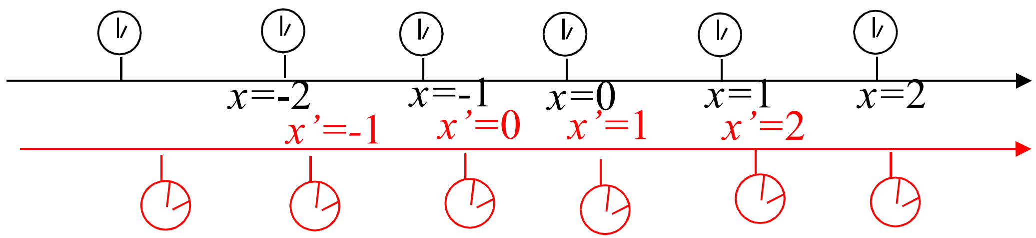
Jakie jeszcze *a* jest możliwe??

Zegary



$$x = Vt - a(V)\,x'$$

$$x' = Vt' - a(V)\,x$$



$$x = Vt + a(V)\,x'$$

$$x' = -Vt' + a(V)\,x$$

$$(x + x')(1 - a) = V(t - t')$$

3Układy

$$x = Vt + a(V)x'$$

$$x' = -Vt' + a(V)x$$

$$x' = v't' + a(v')x''$$

$$x'' = -v't'' + a(v')x'$$

Cd 3 układy

$$x = Vt + a(V)x'$$

$$x' = -Vt' + a(V)x \qquad /\otimes v'$$

$$x' = v't' + a(v')x'' \qquad /\otimes V \qquad +$$

$$x'' = -v't'' + a(v')x'$$

$$a(V) \equiv a$$

$$a(v') \equiv a'$$

$$x'(V+v') = v'a(V)x + Va(v')x''$$

$$x' = \frac{av'x + a'Vx''}{v'+V}$$

1-szy i trzeci

$$x = Vt + a(V)\frac{av'x + a'V\,x''}{v' + V}$$

$$x'' = -v't'' + a(v')\frac{av'x + a'V\,x''}{v' + V}$$

Stała C

uzyskujemy	**musi być**
$$x=\frac{V+v'}{1+\frac{1-a^{2}}{V^{2}}Vv'}\,t+\frac{aa'}{1+\frac{1-a^{2}}{V^{2}}Vv'}\,x''$$	$$x=v\,t+a(v)x''$$
$$x''=-\frac{V+v'}{1+\frac{1-a'^{2}}{v'^{2}}Vv'}\,t''+\frac{aa'}{1+\frac{1-a'^{2}}{v'^{2}}Vv'}\,x$$	$$x''=-v\,t''+a(v)x$$

musi być:

$$\frac{1-a^{2}(V)}{V^{2}}=\frac{1-a^{2}(v')}{v'^{2}}\equiv \mathrm{const}=C$$

wyniki

$$\frac{1-a^2(V)}{V^2}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^2}$$

$$x=Vt+\sqrt{1-CV^2}\,x'$$

$$x'=-Vt'+\sqrt{1-CV^2}\,x$$

$$v=\text{"}v'+V\text{"}=\frac{V+v'}{1+CVv'}$$

wyniki

$$\frac{1-a^2(V)}{V^2}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^2}$$

$$x=Vt+\sqrt{1-CV^2}\,x'$$

$$x'=-Vt'+\sqrt{1-CV^2}\,x \qquad x=\frac{x'+Vt'}{\sqrt{1-CV^2}}$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

wyniki

$$\frac{1-a^2(V)}{V^2}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^2}$$

$$x=Vt+\sqrt{1-CV^2}\,x' \qquad t=\frac{t'+CVx'}{\sqrt{1-CV^2}}$$

$$x'=-Vt'+\sqrt{1-CV^2}\,x \qquad x=\frac{x'+Vt'}{\sqrt{1-CV^2}}$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

Zestawienie z Eukl.

$$\frac{1-a^2(V)}{V^2}=C \quad \Rightarrow \quad a(V)=\sqrt{1-CV^2}$$

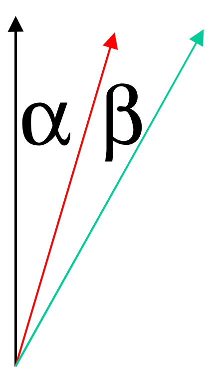
$$x=Vt+\sqrt{1-CV^2}\,x' \qquad x=Ky+\sqrt{1+K^2}\,\boldsymbol{x'}$$

$$x'=-Vt'+\sqrt{1-CV^2}\,x \qquad \boldsymbol{x'=-Ky'}+\sqrt{1+K^2}\,x$$

$$v="v'+V"=\frac{V+v'}{1+CVv'}$$

$$k=\tan(\alpha+\beta)=\frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\cdot\tan(\beta)}=$$

$$=\frac{K+k'}{1-Kk'}$$

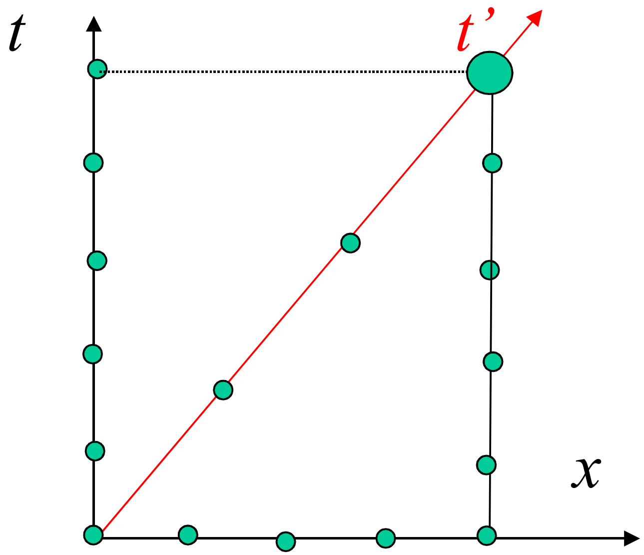


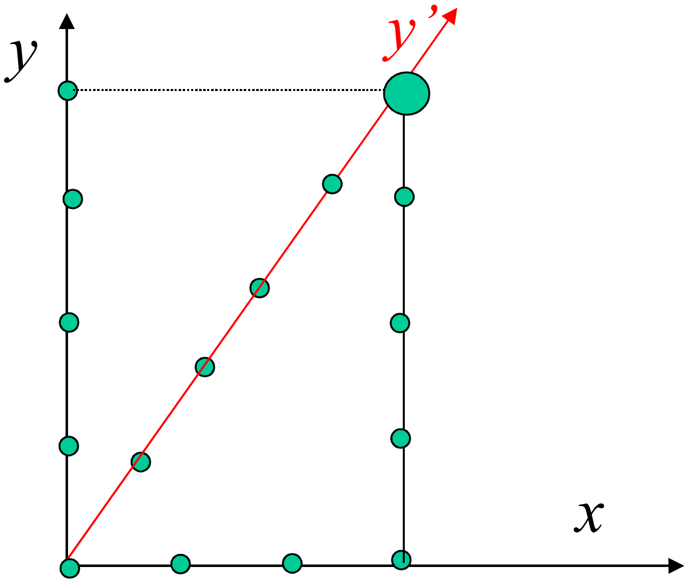
W czasoprzestrzeni

$$x' = 0 \Rightarrow t' = \sqrt{1 - CV^2}\, x / V = \sqrt{t^2 - Cx^2}$$

U Euklidesa

$$x' = 0 \Rightarrow y' = \sqrt{1 + K^2}\, x / K = \sqrt{y^2 + x^2}$$

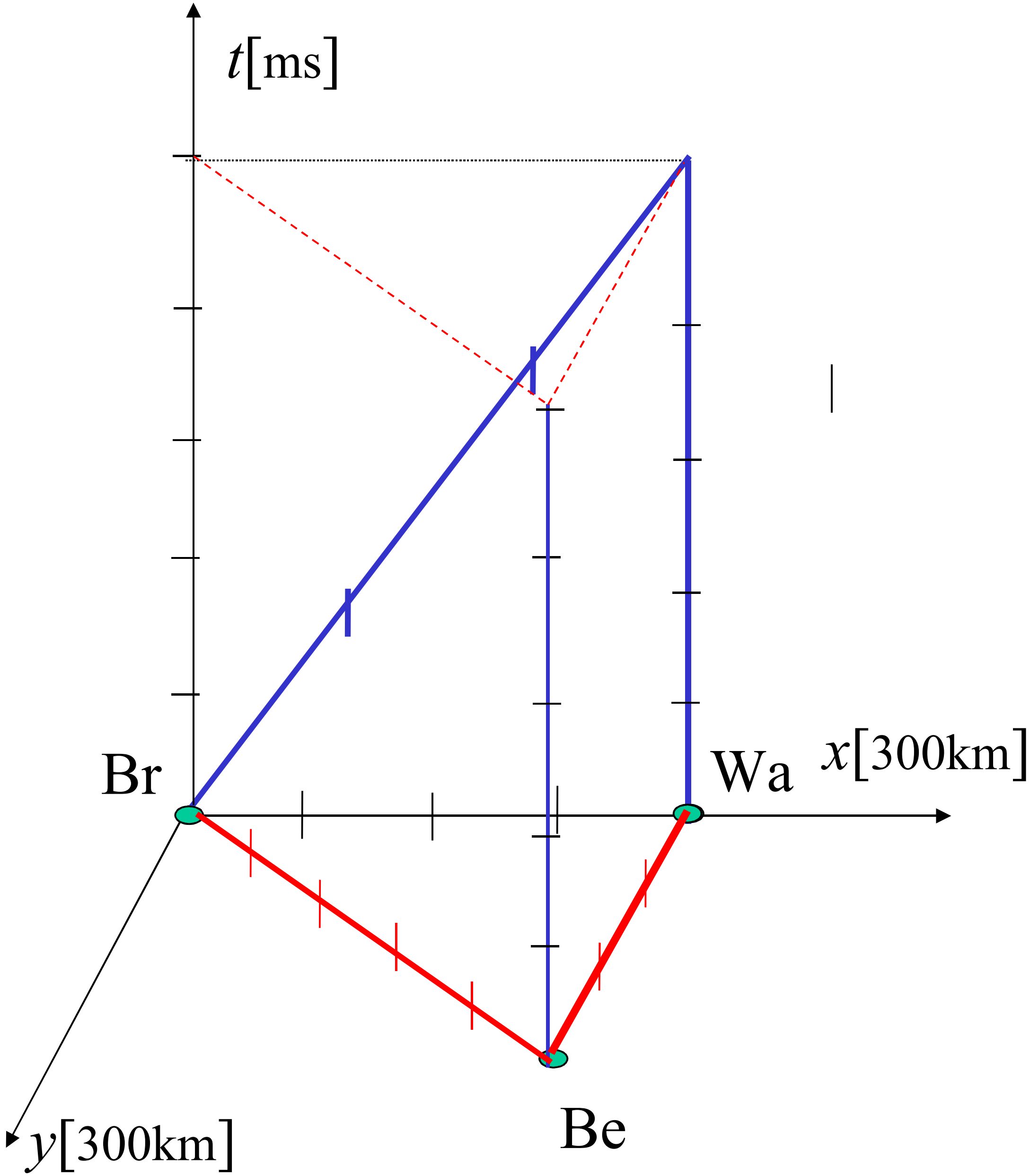




x=4·300km, t=5·1ms, t'=3ms

x=3, y = 4, y'=5

Europa”



c

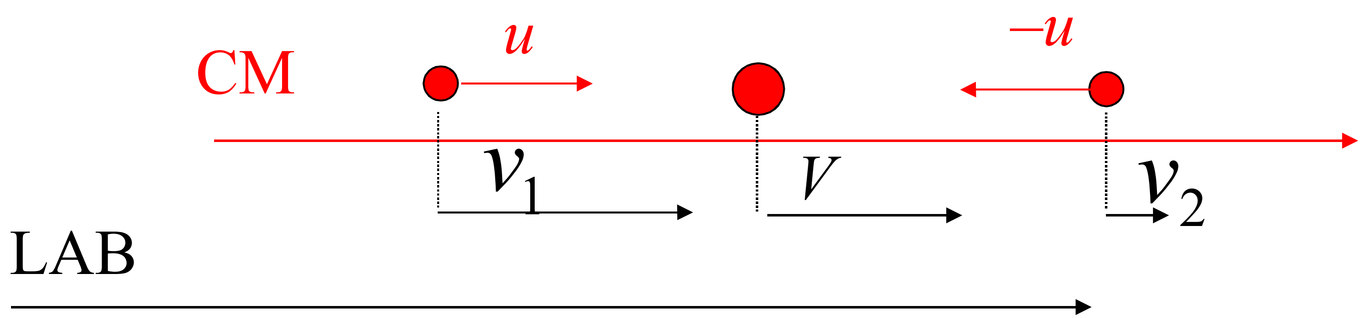
Prędkość *V* nie może być dowolna (minus pod pierwiastkiem):

$$1 - CV^2 \geq 0 \quad \Rightarrow V \leq \frac{1}{\sqrt{C}} = c$$

$$C = 1{,}1 \cdot 10^{-17} \frac{s^2}{m^2} = \frac{1}{(300000 km/s)^2} \equiv \frac{1}{c^2}$$

Jest to prędkość graniczna i zarazem absolutna:

$$"V + c" = \frac{c+V}{1+CVc} = \frac{c+V}{1+Vc/c^2} = c\,\frac{1+V/c}{1+V/c} = c$$



$$v_1 = \frac{V + u}{1 + CVu}, \qquad v_2 = \frac{V - u}{1 - CVu}, \qquad v_3 = V$$

Typowy problem: Znam prędkości początkowe v_1 i v_2 chcę przewidzieć prędkość końcową v_3.

Dygresja:

Zapomnijmy o STW

$$+\quad \begin{aligned} v_1 &= V + u \\ v_2 &= V - u \end{aligned} \qquad\qquad v_3 = V$$

$$2V = 1v_1 + 1v_2 = 2v_3 \qquad /\otimes m$$

$$1 \quad +1 \quad = 2$$

$$mv_1 + mv_2 = m_3 v_3$$

$$m \quad + m \quad = m_3$$

czteroprędkość

$$\frac{1}{\sqrt{1-Cv_{1/2}^{\;2}}}=\frac{1}{\sqrt{1-C\left(\dfrac{V\pm u}{1\pm CVu}\right)^2}}=$$

$$\frac{1\pm CVu}{\sqrt{(1\pm CVu)^2-C(V\pm u)^2}}=\frac{1\pm CVu}{\sqrt{1-CV^2}\sqrt{1-Cu^2}}$$

$$\frac{v_{1/2}}{\sqrt{1-Cv_{1/2}^{\;2}}}=\frac{(1\pm CVu)\dfrac{V\pm u}{1\pm CVu}}{\sqrt{1-CV^2}\sqrt{1-Cu^2}}=\frac{V\pm u}{\sqrt{1-CV^2}\sqrt{1-Cu^2}}$$

$$\frac{v_1}{\sqrt{1-Cv_1^{\,2}}}+\frac{v_2}{\sqrt{1-Cv_2^{\,2}}}=\frac{2}{\sqrt{1-Cu^2}}\,\frac{V}{\sqrt{1-CV^2}}$$

$$V=v_3$$

$$\frac{1}{\sqrt{1-Cv_1^{\,2}}}+\frac{1}{\sqrt{1-Cv_2^{\,2}}}=\frac{2}{\sqrt{1-Cu^2}}\,\frac{1}{\sqrt{1-CV^2}}$$

$$\frac{mv_1}{\sqrt{1-Cv_1^{\,2}}}+\frac{mv_2}{\sqrt{1-Cv_2^{\,2}}}=\frac{m_3v_3}{\sqrt{1-Cv_3^{\,2}}}$$

$$\frac{m}{\sqrt{1-Cv_1^{\,2}}}+\frac{m}{\sqrt{1-Cv_2^{\,2}}}=\frac{m_3}{\sqrt{1-Cv_3^{\,2}}}$$

$$\frac{v_1}{\sqrt{1-Cv_1^{\,2}}}+\frac{v_2}{\sqrt{1-Cv_2^{\,2}}}=\frac{2}{\sqrt{1-Cu^2}}\,\frac{V}{\sqrt{1-CV^2}}$$

$$V=v_3$$

$$\frac{1}{\sqrt{1-Cv_1^{\,2}}}+\frac{1}{\sqrt{1-Cv_2^{\,2}}}=\frac{2}{\sqrt{1-Cu^2}}\,\frac{1}{\sqrt{1-CV^2}}$$

$$\frac{mv_1}{\sqrt{1-Cv_1^{\,2}}}+\frac{mv_2}{\sqrt{1-Cv_2^{\,2}}}=\frac{m_3v_3}{\sqrt{1-Cv_3^{\,2}}}$$

$$\frac{m/C}{\sqrt{1-Cv_1^{\,2}}}+\frac{m/C}{\sqrt{1-Cv_2^{\,2}}}=\frac{m_3/C}{\sqrt{1-Cv_3^{\,2}}}$$

$$p = \frac{mv}{\sqrt{1 - Cv^2}}$$

$$\frac{m/C}{\sqrt{1 - Cv^2}} = m/C + \left(\frac{m/C}{\sqrt{1 - Cv^2}} - m/C \right) = m/C + \frac{mv^2}{1 - Cv^2 + \sqrt{1 - Cv^2}}$$

$$T = \frac{m/C}{\sqrt{1 - Cv^2}} - m/C$$

$$T + m/C = T + E_{\text{wewn}} \equiv E = \frac{m/C}{\sqrt{1 - Cv^2}}$$

$$p = \frac{mv}{\sqrt{1 - Cv^2}}$$

$$E = \frac{m / C}{\sqrt{1 - Cv^2}}$$

$$v = \frac{p}{EC}$$

$$E = \sqrt{m^2 / C^2 + p^2 / C}$$

mc^2

$$\sum_{\text{konc}} \left(T + \frac{m}{C}\right) = \sum_{\text{pocz.}} \left(T + \frac{m}{C}\right)$$

$$\sum_{\text{konc}} T - \sum_{\text{pocz.}} T = -\frac{\sum_{\text{konc}} m - \sum_{\text{pocz.}} m}{C} = \sum_{\text{pocz.}} mc^2 - \sum_{\text{konc}} mc^2 = c^2 \Delta m$$