

Homework (Lecture 1)

1. For x, y, z and their uncertainties $\sigma_x, \sigma_y, \sigma_z$:

$$x \pm \sigma_x = (2.0 \pm 0.1) \text{ cm}$$

$$y \pm \sigma_y = (6.0 \pm 0.2) \text{ cm}$$

$$z \pm \sigma_z = (1.65 \pm 0.01) \text{ cm}$$

1a) Find $f = (x + y - 2z)$ and its uncertainty σ_f assuming that x, y and z are uncorrelated.

1b) Find $f = (x + y - 2z)$ and its uncertainty σ_f assuming that x, y and z are correlated and the correlation coefficients are: $\rho_{xy}=1, \rho_{xz} = \rho_{yz} = 0.5$.

2. For x, z and their uncertainties σ_x, σ_z :

$$x \pm \sigma_x = (2.0 \pm 0.1) \text{ cm}$$

$$z \pm \sigma_z = (1.65 \pm 0.01) \text{ cm}$$

2a) Find $f = xz$ and its uncertainty σ_f assuming that x, z are uncorrelated.

2b) Find $f = xz$ and its uncertainty σ_f assuming that x, z are correlated and the correlation coefficient is $\rho_{xz} = 0.5$

Homework (Lecture1) cont'ed

3. For x, y and their uncertainties σ_x, σ_y :

$$x \pm \sigma_x = (2.0 \pm 0.2) \text{ sec}$$

$$y \pm \sigma_y = (3.0 \pm 0.6) \text{ sec}$$

3a) Find $f = x/y$ and its uncertainty σ_f assuming that x, y are uncorrelated.

3b) Find $f = x/y$ and its uncertainty σ_f assuming that x, y are fully correlated (the correlation coefficient is $\rho_{xy} = 1$)

4. For x, y and their uncertainties σ_x, σ_y :

$$x \pm \sigma_x = (4.0 \pm 0.1) \text{ cm}$$

$$y \pm \sigma_y = (2.0 \pm 0.2) \text{ cm}$$

4a) Find $f = y^2/\sqrt{x}$ and its uncertainty σ_f assuming that x, y are uncorrelated.

4b) Find $f = y^2/\sqrt{x}$ and its uncertainty σ_f assuming that x, y are fully anti-correlated (the correlation coefficient is $\rho_{xy} = -1$)

4c) Find $f = zx + y^2$ and its uncertainty σ_f assuming that x, y and z are uncorrelated.

Homework (Lecture1) cont'ed

5. Find $f = \sin(\vartheta)$ and its uncertainty σ_f for $\theta \pm \sigma_\theta = (53 \pm 2)^\circ$

6. For x, ϑ and their uncertainties $\sigma_x, \sigma_\vartheta$:

$$x \pm \sigma_x = (2.0 \pm 0.2) \text{ cm}$$

$$\theta \pm \sigma_\theta = (53 \pm 2)^\circ$$

Find $f = x \cos(\vartheta)$ and its uncertainty σ_f assuming that x and ϑ are uncorrelated.

7. The radius of a very thin disk is $r \pm \sigma_r = (10.0 \pm 0.1) \text{ cm}$.

7a) Find the circumference and the area of this disk and their uncertainties.

7b) What is the relative uncertainty on r ? Is the relative uncertainty on the circumference and the area different than the relative uncertainty on r ?

7c) Given a (hypothetical) measuring tape with a total length of 180 cm and with an absolute measurement uncertainty of 0.1 cm at any position along its length, discuss methods to improve the determination of the disk's radius of $r \pm \sigma_r = (10.0 \pm 0.1) \text{ cm}$, including a discussion of their uncertainties. Hint: systematic uncertainty discussion would include the discussion of the thickness of the tape and/or the disk itself.