Zad 0.

1

a) $\frac{1-x^3}{1-x^5} = f(x)$, $x \neq 1$

$$f'(x) = \frac{(1-x^3)'(1-x^5)-(1-x^5)'(1-x^3)}{(1-x^5)^2} = \frac{-3x^2(1-x^5)+5x^4(1-x^3)}{(1-x^5)^2} =$$

$$= \frac{-3x^2+3x^7+5x^4-5x^7}{(1-x^5)^2} = \frac{-2x^7+5x^4-3x^2}{(1-x^5)^2}$$

b) $f(x) = \frac{2}{(1-x^2)(1+x^4)}$

$$f'(x) = \frac{-2[(1-x^2)(1+x^4)]'}{[(1-x^2)(1+x^4)]^2} = \frac{-2[-2x(1+x^4)+4x^3(1-x^2)]}{[(1-x^2)(1+x^4)]^2} =$$

$$= \frac{-2(-2x-2x^5+4x^3-4x^5)}{[(1-x^2)(1+x^4)]^2} = \frac{4x^2+4x^5-8x^3+8x^5}{[(1-x^2)(1+x^4)]^2} =$$

$$= \frac{12x^5-8x^3+4x}{[(1-x^2)(1+x^4)]^2}$$

$(\operatorname{arc\,tg} x)' = \frac{1}{1+x^2}$, $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

c) $f(x) = \frac{\operatorname{arc\,tg} x}{\arcsin x}$

$$f'(x) = \frac{(\operatorname{arc\,tg} x)' \cdot \arcsin x - (\arcsin x)' \operatorname{arc\,tg} x}{(\arcsin x)^2} = \frac{\frac{\arcsin x}{1+x^2} - \frac{\operatorname{arc\,tg} x}{\sqrt{1-x^2}}}{(\arcsin x)^2} =$$

$$= \frac{1}{(\arcsin x)^2}\left(\frac{\sqrt{1-x^2}\arcsin x - (1+x^2)\operatorname{arc\,tg} x}{(1+x^2)\sqrt{1-x^2}}\right) =$$

$$= \frac{\cancel{\sqrt{1-x^2}}\,\cancel{\arcsin x}}{(1+x^2)\cancel{\sqrt{1-x^2}}(\arcsin x)^{\cancel{2}}} - \frac{\cancel{(1+x^2)}\operatorname{arc\,tg} x}{\cancel{(1+x^2)}\sqrt{1-x^2}(\arcsin x)^2} =$$

$$= \frac{1}{(1+x^2)\arcsin x} - \frac{\operatorname{arc\,tg} x}{\sqrt{1-x^2}(\arcsin x)^2}$$

d) $f(x) = e^x(\sin x + \cos x)$ $\qquad (e^x)' = e^x$

$$f'(x) = e^x(\sin x+\cos x) + e^x(\cos x - \sin x) = e^x(\cancel{\sin x} + \cos x + \cos x - \cancel{\sin x}) =$$

$$= 2e^x \cos x$$

e) $f(x) = \sin^4 5x$

$f'(x) = 4\sin^3 5x \cdot \cos 5x \cdot 5 = 20 \sin^3 5x \cos 5x$

f) $f(x) = \log\left(x + \sqrt{x^4+4}\right)$

$f'(x) = \frac{1}{\left(x+\sqrt{x^4+4}\right)} \cdot \left(1 + \frac{\cancel{2}\cancel{4}x^3}{\cancel{2}\sqrt{x^4+4}}\right) = \frac{1}{x+\sqrt{x^4+4}} + \frac{2x^3}{\sqrt{x^4+4}\left(x+\sqrt{x^4+4}\right)} =$

$= \frac{1}{x+\sqrt{x^4+4}} + \frac{\frac{2x^3}{\sqrt{x^4+4}}}{x+\sqrt{x^4+4}} = \frac{1+\frac{2x^3}{\sqrt{x^4+4}}}{x+\sqrt{x^4+4}}$

g) $f(x) = \log(\log(\log x))$

$f'(x) = \frac{1}{x} \cdot \frac{1}{\log x} \cdot \frac{1}{\log(\log x)} = \frac{1}{x \log x \cdot \log(\log x)}$

h) $f(x) = \sin(\cos x) + \cos(\sin x)$

$f'(x) = -\sin x \cdot \cos(\cos x) - \cos x \cdot \sin(\sin x)$

i) $f(x) = 2^{\operatorname{tg}\frac{1}{x}}$

$a^x = a^x \ln a$

$f'(x) = -\frac{1}{x^2} \cdot \frac{1}{\cos^2\left(\frac{1}{x}\right)} \cdot 2^{\operatorname{tg}\frac{1}{x}} \cdot \log[2] = -\frac{2^{\operatorname{tg}\frac{1}{x}} \cdot \log[2]}{x^2 \cos^2\left(\frac{1}{x}\right)}$

j) $f(x) = e^{\text{arc tg}^3 \sqrt{x+4}}$

$$f'(x) = e^{\text{arc tg}^3 \sqrt{x+4}} \cdot \left(\text{arc tg}^3 \sqrt{x+4}\right)' = e^{\text{arc tg}^3\sqrt{x+4}} \cdot \frac{1}{2\sqrt{x+4}} \cdot \frac{1}{1+(\sqrt{x+4})^2} \cdot$$

$$\cdot\, 3\,\text{arc tg}^2 \sqrt{x+4} = \frac{3\,\text{arc tg}^2\sqrt{x+4}}{2\sqrt{x+4}\,(1+x+4)}\, e^{\text{arc tg}^3\sqrt{x+4}} =$$

$$= \frac{3\,\text{arc tg}^2\sqrt{x+4}}{2\sqrt{x+4}\cdot(5+x)}\, e^{\text{arc tg}^3\sqrt{x+4}}$$

k) $f(x) = \log^2\left(\text{arc sin}^3 \sqrt{x}\right)$

$$f'(x) = \cancel{2}\log\left(\text{arc sin}^3\sqrt{x}\right) \cdot \frac{1}{\text{arc sin}^{\cancel{3}}\sqrt{x}} \cdot \frac{1}{\sqrt{1-(\sqrt{x})^2}} \cdot 3\,\cancel{\text{arc sin}^2}\sqrt{x} \cdot \frac{1}{\cancel{2}\sqrt{x}} =$$

$$= \frac{3\log\left(\text{arc sin}^3\sqrt{x}\right)}{\sqrt{x(1-x)}\,\text{arc sin}\sqrt{x}}$$

l) $f(x) = \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$

$$f'(x) = \frac{1}{\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}} \cdot \left(-\frac{1}{x^2} + \frac{1}{\cancel{2}\sqrt{1+\frac{1}{x^2}}} \cdot \left(-\frac{\cancel{2}x}{x^{\cancel{4}3}}\right)\right) = \frac{-\frac{1}{x^2} - \frac{1}{x^3\sqrt{1+\frac{1}{x^2}}}}{\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}} =$$

$$= \frac{-\frac{1}{x^2}\left(1+\frac{1}{x\sqrt{1+\frac{1}{x^2}}}\right)}{\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}} = \frac{-\frac{1}{x^2}\left(\frac{x\sqrt{1+\frac{1}{x^2}}+1}{x\sqrt{1+\frac{1}{x^2}}}\right)}{\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}} = -\frac{1}{x^2}\cdot\left(\frac{x\sqrt{1+\frac{1}{x^2}}+1}{x\sqrt{1+\frac{1}{x^2}}}\right)\cdot\frac{1}{\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}}$$

$$= -\frac{x\sqrt{1+\frac{1}{x^2}}+1}{x^2\cdot x\sqrt{1+\frac{1}{x^2}}\cdot\left(\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)} = -\frac{x\sqrt{1+\frac{1}{x^2}}+1}{x\sqrt{1+\frac{1}{x^2}}\cdot\left(x+x^2\sqrt{1+\frac{1}{x^2}}\right)} = \frac{-\cancel{\left(x\sqrt{1+\frac{1}{x^2}}+1\right)}}{x\sqrt{1+\frac{1}{x^2}}\cdot x\cancel{\left(1+x\sqrt{1+\frac{1}{x^2}}\right)}}$$

$$= -\frac{1}{x^2\sqrt{1+\frac{1}{x^2}}}$$

m) $f(x) = \sinh^3 4x$ $\qquad (\sinh x)' = \cosh x$

$$f'(x) = 3\sinh^2 4x \cdot \cosh 4x \cdot 4 = 12\cosh 4x \sinh^2 4x$$

e) $f(x) = \sin^4 5x$

$$f'(x) = 4\sin^3 5x \cdot \cos 5x \cdot 5 = 20\sin^3 5x \cos 5x$$

f) $f(x) = \log\left(x + \sqrt{x^4+4}\right)$

$$f'(x) = \frac{1}{\left(x+\sqrt{x^4+4}\right)} \cdot \left(1 + \frac{2\cancel{4}x^3}{\cancel{2}\sqrt{x^4+4}}\right) = \frac{1}{x+\sqrt{x^4+4}} + \frac{2x^3}{\sqrt{x^4+4}\left(x+\sqrt{x^4+4}\right)} =$$

$$= \frac{1}{x+\sqrt{x^4+4}} + \frac{\frac{2x^3}{\sqrt{x^4+4}}}{x+\sqrt{x^4+4}} = \frac{1+\frac{2x^3}{\sqrt{x^4+4}}}{x+\sqrt{x^4+4}}$$

g) $f(x) = \log(\log(\log x))$

$$f'(x) = \frac{1}{x} \cdot \frac{1}{\log x} \cdot \frac{1}{\log(\log x)} = \frac{1}{x\log x \cdot \log(\log x)}$$

h) $f(x) = \sin(\cos x) + \cos(\sin x)$

$$f'(x) = -\sin x \cdot \cos(\cos x) - \cos x \cdot \sin(\sin x)$$

i) $f(x) = 2^{tg\frac{1}{x}}$ $\qquad a^x = a^x \ln a$

$$f'(x) = -\frac{1}{x^2} \cdot \frac{1}{\cos^2\left(\frac{1}{x}\right)} \cdot 2^{tg\frac{1}{x}} \cdot \log[2] = -\frac{2^{tg\frac{1}{x}} \cdot \log[2]}{x^2\cos^2\left(\frac{1}{x}\right)}$$

5

s) $f(x)=(x+1)^{\frac{1}{\sin x}}$

$$f'(x)=\left\{x+1=e^{\log(x+1)}\right\}=\left(e^{\frac{\log(x+1)}{\sin x}}\right)'=e^{\frac{\log(x+1)}{\sin x}}\cdot\frac{\frac{1}{x+1}\sin x-\cos x\cdot\log(x+1)}{\sin^2 x}=$$

$$=e^{\frac{\log(x+1)}{\sin x}}\cdot\frac{\frac{\sin x}{x+1}-\cos x\cdot\log(x+1)}{\sin^2 x}=(x+1)^{\frac{1}{\sin x}}\cdot\frac{\cancel{\sin^2 x}\left(\frac{1}{(x+1)\sin x}-\frac{\operatorname{ctg}x}{\sin x}\log(x+1)\right)}{\cancel{\sin^2 x}}=$$

$$=(x+1)^{\frac{1}{\sin x}}\left(\frac{1}{(x+1)\sin x}-\frac{\operatorname{ctg}x}{\sin x}\log(x+1)\right)$$

t) $f(x)=x^{\sin x}$

$$f'(x)=\left\{x=e^{\log x}\right\}=\left(e^{\log x\cdot\sin x}\right)'=e^{\log x\cdot\sin x}\cdot\left(\frac{1}{x}\cdot\sin x+\cos x\cdot\log x\right)=$$

$$=x^{\sin x}\left(\frac{\sin x}{x}+\cos x\log x\right)$$

u) $f(x)=x^{x^2}$

$$f'(x)=\left\{x=e^{\log x}\right\}=\left(e^{x^2\log x}\right)'=e^{x^2\log x}\cdot\left(2x\cdot\log x+\frac{1}{\cancel{x}}\cdot x^{\cancel{2}}\right)=x^{x^2}(2x\log x+x)$$

w) $f(x)=x^{x^x}$

$$f'(x)=\left\{x=e^{\log x}\right\}=\left(e^{\log x\cdot x^x}\right)'=\left(e^{\log x\cdot e^{\log x\cdot x}}\right)'=e^{\log x\cdot e^{\log x\cdot x}}\left(\frac{1}{x}\cdot e^{\log x\cdot x}+\right.$$

$$+\log x\cdot e^{\log x\cdot x}\cdot\left(\frac{1}{\cancel{x}}\cdot\cancel{x}+\log x\right)=e^{\log x\cdot e^{\log x\cdot x}}\cdot\left(\frac{e^{\log x\cdot x}}{x}+\log x\cdot e^{\log x\cdot x}+\right.$$

$$\left.+(\log x)^2\cdot e^{\log x\cdot x}\right)=x^{x^x}\left(\frac{x^x}{x}+\log x\cdot x^x+(\log x)^2\cdot x^2\right)$$

x) $f(x)=\log_2(x^4+1)$

$$\cancel{f'(x)=\left(\frac{\ln(x^4+1)}{\ln 2}\right)'=\frac{1}{x^4+1}\cdot 4x^3}$$

$$(\log_a x)'=\left(\frac{\ln x}{\ln a}\right)'=\frac{\frac{1}{x}\ln a-\frac{1}{a}\ln x}{(\ln a)^2}$$

$$(\log_a x)'=\frac{1}{x\ln a}$$

$$f'(x)=\frac{1}{(x^4+1)\ln 2}\cdot 4x^3$$

6

y) $f(x)=\log_x(x^4+1)$

$$f'(x)=\left(\frac{\ln(x^4+1)}{\ln x}\right)'=\frac{\frac{1}{x^4+1}\cdot 4x^3\cdot\ln x-\frac{1}{x}\ln(x^4+1)}{(\ln x)^2}=$$

$$=\left[\frac{4x^3\cdot\ln x}{x^4+1}-\frac{1}{x}\ln(x^4+1)\right]\cdot\frac{1}{(\ln x)^2}=$$

$$=\frac{4x^3}{(x^4+1)\ln x}-\frac{\ln(x^4+1)}{x(\ln x)^2}$$

z) $f(x)=\sqrt[3]{x^2}\sin x\log x$

$(uvw)'=u'vw+uv'w+uvw'$

$(\sqrt[n]{x})'=\frac{1}{n\sqrt[n]{x^{n-1}}}$

$$\cancel{f'(x)=\frac{2x}{3\sqrt[3]{(x^2)^2}}\sin x\log x+\sqrt[3]{x^2}\cos x\log x+\sqrt[3]{x^2}\sin x\frac{1}{x}=}$$

$$\cancel{=\frac{2x\sin x\log x}{3(x^2)^{2/3}}+\cos x\log x(x)^{2/3}+\frac{(x)^{2/3}\sin x}{x}}$$

z:

$$\left(x^{2/3}\sin x\log x\right)'=\frac{2}{3}x^{-1/3}\sin x\log x+$$

$$x^{2/3}\cos x\log x+x^{2/3}\sin x\,\frac{1}{x}=$$

$$=x^{-1/3}\left(\tfrac{2}{3}\sin x\log x+x\cos x\log x+\sin x\right)$$

$$\sqrt[n]{x}=x^{\frac{1}{n}}\qquad\left(x^{\frac{1}{n}}\right)'=\frac{1}{n}x^{\frac{1}{n}-1}=\frac{1}{n\sqrt[n]{x^{n-1}}}$$