Najprościej będzie przyjrzeć się rozwinięciom w szereg

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{5!}x^5$$

$$\log\left(\frac{x}{\sin x}\right) = -\log\left(\frac{\sin x}{x}\right) = -\log\left(1 - \frac{1}{6}x^2 + \frac{1}{120}x^4 - \dots\right) =$$

$$= -\left[\left(-\frac{1}{6}x^2 + \frac{1}{120}x^4 - \dots\right) - \frac{1}{2}\left(-\frac{1}{6}x^2 + \frac{1}{20}x^4 - \dots\right)^2 + \dots\right) =$$

$$= -\left[-\frac{1}{6}x^2 + \frac{1}{120}x^4 - \frac{1}{2}\left(\frac{1}{36}x^4 + \dots\right)\right] =$$

$$= -\left[\frac{1}{6}x^2 + \underbrace{\left(\frac{1}{120} - \frac{1}{72}\right)}_{-1/180}x^4 + \dots\right] =$$

$$= \frac{1}{6}x^2 + \frac{1}{180}x^4 + \dots$$

$$\frac{6}{x^2}\log\left(\frac{x}{\sin x}\right) = 1 + \frac{1}{30}x^2 + \dots$$

$$\exp\left(\frac{6}{x^2}\log\left(\frac{x}{\sin x}\right)\right) = \exp\left(1 + \frac{1}{30}x^2 + \dots\right) =$$

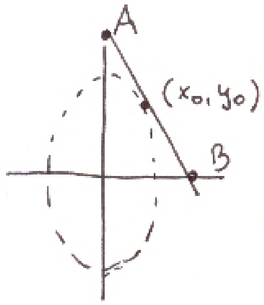
$$= e \cdot \exp\left(1/30\, x^2 + \dots\right) = e\left(1 + \frac{1}{30}x^2 + \dots + \frac{1}{2}\left(\frac{1}{30}x^2 \dots\right)^2\right)$$

$$= e\left(1 + \frac{1}{30}x^2\right)$$

Pochodna istnieje i jest równa zero!

$$\lim \frac{1}{x}\left(e + \frac{e}{30}x^2 + \dots - e\right) = 0$$

Zadanie 7:



$$1 = \frac{x_0^2}{8} + \frac{y_0^2}{18}$$

$$y_0^2 = 18\left(1 - \frac{x_0^2}{8}\right) = 9\left(2 - \frac{x_0^2}{4}\right) =$$

$$= \frac{9}{4}\left(8 - x_0^2\right)$$

$$y_0 = \frac{3}{2}\sqrt{8 - x_0^2}$$

$$y_0'(x_0) = \frac{3}{2}\,\frac{1}{2\sqrt{8 - x_0^2}} \cdot (-2x_0) = -\frac{3x_0}{2\sqrt{8 - x_0^2}}$$

Styczna ma równanie

$$y = -\frac{3x_0}{2\sqrt{8 - x_0^2}}\,x + b$$

b wyznaczamy z warunku, że (x_0, y_0) należy do prostej:

$$\frac{3}{2}\sqrt{8 - x_0^2} + \frac{3x_0^2}{2\sqrt{\quad}} = b$$

$$\frac{3}{2} \cdot \frac{1}{\sqrt{\quad}}\left(8 - x_0^2 + 3x_0^2\right) = b$$

$$b = \frac{3}{2\sqrt{\quad}}\left(8 + 2x_0^2\right) = \frac{3\left(4 + x_0^2\right)}{2\sqrt{8 - x_0^2}}$$

$$y = -\frac{3x_0}{2\sqrt{8 - x_0^2}}\,x + \frac{3\left(4 + x_0^2\right)}{2\sqrt{8 - x_0^2}}$$

A: $\left(0, \frac{3(4+x_0^2)}{2\sqrt{8-x_0^2}}\right)$

B:

$$0 = -\frac{3x_0}{2\sqrt{\quad}}\, x + \frac{3(4+x_0^2)}{2\sqrt{\quad}}$$

$$x = \frac{3(4+x_0^2)}{2\sqrt{\quad}} \cdot \frac{2\sqrt{\quad}}{3x_0} = \frac{4+x_0^2}{x_0}$$

$$\left(\frac{4+x_0^2}{x_0}, 0\right)$$

$$P(x_0) = \frac{1}{2} \cdot \frac{3(4+x_0^2)}{2\sqrt{\quad}} \cdot \frac{(4+x_0)^2}{x_0} = \frac{3}{4} \frac{(4+x_0^2)^2}{x_0\sqrt{8-x_0^2}}$$

$$\lim_{x_0 \to 0} P(x_0) = \infty \qquad \lim_{x_0 \to 2\sqrt{2}} P(x_0) = \infty$$

$$P'(x_0) = \frac{3}{4} \frac{2(4+x_0^2)\cdot 2x_0^2\sqrt{\quad} - \left(\sqrt{\quad} + \frac{x_0\cdot(-\cancel{2}x_0)}{\cancel{2}\sqrt{\quad}}\right)(4+x_0^2)^2}{x_0^2(8-x_0^2)}$$

$$= \frac{3}{4} \frac{1}{x_0^2(8-x_0^2)} \underbrace{\left[4x_0^2(8-x_0^2) - \{8-x_0^2 - x_0^2\}(4+x_0^2)\right]}_{} (4+x_0^2)$$

$$32x_0^2 - 4x_0^4 - (8-2x_0^2)(4+x_0^2) =$$

$$= 32x_0^2 - 4x_0^4 - 32 + 8x_0^2 - 8x_0^2 + 2x_0^4 =$$

$$= -2x_0^4 + 32x_0^2 - 32 =$$

$$= -2(x_0^4 \cancel{+} - 16x_0^2 + 16)$$

$$\Delta = (16)^2 - 4\cdot 16 =$$

$$= 16\cdot 4[4-1] =$$

$$= 16\cdot 4\cdot 3$$

$$\sqrt{\Delta} = 4\cdot 2\sqrt{3} = 8\sqrt{3}$$

$$x_0^2 = \frac{16 \pm 8\sqrt{3}}{2} = 8 \pm 4\sqrt{3} \qquad \sqrt{8+4\sqrt{3}} > 2\sqrt{2}$$

interesuje nas tylko

$$\sqrt{8-4\sqrt{3}}$$

okazuje się że

$$\sqrt{8-4\sqrt{3}} = \sqrt{6} - \sqrt{2}$$

$$x_0 = \sqrt{6} - \sqrt{2}$$

$$y_0 = \frac{3}{2}\sqrt{8-(8-4\sqrt{3})} = \frac{3}{2}\sqrt{8-8+4\sqrt{3}} = \frac{3}{2}\sqrt{4\sqrt{3}} =$$
$$= 3\sqrt[4]{3}.$$

punkt: $\left(\sqrt{6}-\sqrt{2},\ 3\sqrt[4]{3}\right)$