4 marca 2011
17:40

D(12345) = \( a_1 \cdot a_2 \cdot a_3 \cdot a_4 \cdot a_5 \)

\[
\begin{align*}
(2345) & \times M^2, 23, 235, 234 \\
(345) & \times M^3, 135, 135, 134 \\
(425) & \times M^4, 235, 235, 235, 234 \\
(125) & \times M^5, 235, 135, 135, 134 \\
(1234) & \times M^6, 234, 134, 134, 134
\end{align*}
\]

\[3 \rightarrow 1 \rightarrow 5 \rightarrow 7 \rightarrow 4 \rightarrow 1\]

\[
\begin{align*}
\text{Vertex : gluing of } D+1 & \text{ (D) simplex into } \\
& \text{ into a } D \text{ simplex}
\end{align*}
\]

\[\sum_{\sigma \text{ even}} \delta_{j_1, i_\sigma(j_1)} \ldots \delta_{j_d, i_\sigma(j_d)}
\]

\[\text{any 6FT graph describes a gluing of } \]

any 6FT graph describes a gluing of
D simplexes along the edges of the graph

Closed loops correspond to D-2 simplices (Regge)

- 2-complex which the 2-skeleton dual to a gluing of D-simplices

- genus object which generalizes that dual of triangulation of manifolds

- $\Pi$ is real and invariant under all $\sigma$

- manifold oriented object

- $\Pi$ is real and invariant under even $\sigma$

- oriented object with twist in the gradient part of the action

See di Pietri, Freidel, Krasnov, Rovelli

"BC model from a BO field over homogenous space" hep-th/9907137

III. BF Theory

Discretization of BF Theory

$$L = \int DA DB e^{i \mathcal{S}(BF)}$$

- $D$-2 form

- $F = DA + A^2$

- $A$: lie algebra valued 1-form

D manifold $\rightarrow$ triangulation

B lives on the faces of the dual of the
\[ Z'' = \int_{\text{surj}} T \sum_{e \in E} \left( T \sum_{f \in E^*} \varepsilon_{f e} \right) \]

\[ A \rightarrow g_{e, e'} \text{ such that edges } e, e' \text{ are in } \Gamma \]

\[ \varepsilon_{f e} = \left\{ \begin{array}{cl} 1 & \text{if the orientation of } f \text{ and } e \text{ agree} \\ -1 & \text{otherwise} \end{array} \right. \]

\[ g_{v, e} g_{v', e} = g_{v, e} \quad \text{s.t. } \text{some and} \]

\[ g_{v, e} \rightarrow g_{v, e} g^{-1}_{v', e} \quad \text{target} \]

\[ Z = \int_{\text{surj}(v)} \prod_{\text{in} v} \delta \left( \prod \sum_{\text{out} v} \varepsilon_{v, v'} g_{v, e} \right) \]

\[ e = \text{edge entering } v \]

\[ e' = \text{edge leaving } v \]

**Group field theory**

\[ \psi(h_0, h_1, \ldots, h_D) \text{ field defined on } \mathcal{G}(a, v) \]

\[ h_0 \rightarrow h'_0 \quad \sum_{\sigma \text{ even}} \delta(h_0, h'_0) \quad \delta(h_0, h'_0) \]

\[ \text{group elements are just indices of the generalised matrix model} \]

\[ \bar{V} \left( h_{ab} \right) \]

\[ a \rightarrow \prod \prod \int d g_{a} \delta \left( h_{ab} g_{a} g_{b}^{-1} h_{ba} \right) \]

\[ a \leq c < b \leq d \]
\[ V(\mathbf{q}) = \frac{1}{D} \sum_{\mathbf{h}} V(\mathbf{h}) \Phi(\mathbf{h}) D^h \] 

Kernel of the interaction

\[ \text{divergence of GFT graphs} \]

\[ \delta(n) \to \sum_{\mathbf{h}} \delta(\mathbf{h}, \mathbf{h}_n) \delta(\mathbf{h}, \mathbf{h}_n^*) \delta(\mathbf{h}_n, \mathbf{h}_n^*) \]

3D graph with 8 cancelation
Two recent breakthroughs

+ $1/N$ expansion of colored tensor model
  R. Gurau 1002.5259

+ Formulation using colored products
  A. Bardini, F. Gliozzi, D. Onofri
  "Differential in group Fredkens'"  
  e104.0590

\[\Delta_\gamma = \sum_{\Delta_{\gamma}} \delta(t') \frac{73}{v} \Delta V(t') (h_{\gamma v})\]  

\[\Delta V(t') = \sum_{\Delta_{\gamma}} \delta(t') \frac{73}{v} \Delta V(t') (h_{\gamma v})\]  

\[\kappa(h, \gamma) = \sum_{\Delta_{\gamma}} \delta(h, \gamma) \frac{73}{v} \Delta V(t') (h_{\gamma v})\]  

\[V_{v_{1}v_{2}} = \sum_{\gamma} V_{v_{1}v_{2}} \text{ as an SU(3)}\]  

\[\text{Tr}_{V_3} (h_{\gamma v}) \text{ Tr}_{V_3} (h_{\gamma v}) (\mathbf{k}) = \begin{cases} 1 & \text{for } v_{1} = v_{2} \\
0 & \text{otherwise} \end{cases}\]  

\[\sum_{\Delta_{\gamma}} \left< h_{\Delta_{\gamma}} \right| h_{\gamma} \left| \Delta_{\gamma} \right> \left< \Delta_{\gamma} \right| h_{\gamma} \left| \Delta_{\gamma} \right>\]  

\[\sum_{\Delta_{\gamma}} \left< h_{\Delta_{\gamma}} \right| h_{\gamma} \left| \Delta_{\gamma} \right> \left< \Delta_{\gamma} \right| h_{\gamma} \left| \Delta_{\gamma} \right>\]
\[ m^{ui} = m^i \quad \gamma^j = \gamma \]

\[ \frac{1}{d_j} \sum_{m, m'} \langle \varphi| \varphi \rangle \]

\[ K_{i,j} = \sum_j \frac{d_j}{V_j} \text{Tr} \left( \tilde{\gamma} \gamma_{\mu} \gamma_{\nu} \right) \]

\[ P_{\tilde{\gamma} \gamma_{\mu} \gamma_{\nu}} \rightarrow V_j \]

\[ \text{SL}(2, \mathbb{C}) \quad \text{SU}(6) \quad \text{rep} \quad \text{rep} \]

\[ K_{i,j} \rightarrow v_{i,j} \]

\[ \delta \left( \prod_{\text{rest}} \right) = \sum_j \frac{d_j}{V_j} \text{Tr} \left( \prod_{\text{rest}} \right) \]

Perform the integration over \( \delta \left( \prod_{\text{rest}} \right) \)

Using Schur's orthogonality relation

\[ \mathcal{A}_4 = \sum_j \frac{d_j}{V_j} \left[ \prod_{\text{rest}} P_{\tilde{\gamma} \gamma_{\mu} \gamma_{\nu}} \right] \]

with insertion of extra \( \delta \left( \prod_{\text{rest}} \right) \)

for the boundary of the 2-cycles

GPT for the EPRL model

\[ \chi \left( h^1, h^2, h^3, h^4 \right) \quad \chi \in \text{SU}(2) \]

Propagator as in BF

Vertex slightly different from BF

\[ \frac{1}{|a < b < c |} \sum_{j} \text{Tr} \left( \tilde{\gamma} \gamma_{\mu} \gamma_{\nu} \right) \]

\[ \left( h^1, h^2, h^3, h^4 \right) \quad \chi \in \text{SU}(2) \]
Correlating junction with $N$ external legs equipped with fields $y_j (h_j, -h_j)$ depends on such group elements can be related to elements of the LQG Hilbert space

If GFT is physical, its prescription for summing Feynman graphs into correlating junction must have same consequences in LQG

Ex: Schwinger–Dyson Equation

**Wilson renormalization of GFT**

- Renormalization:
  - $s(\Delta \lambda, \Delta \phi) \to s(\Delta \lambda, \Delta \phi) - \Delta \lambda, \Delta \phi$
  - $s(\Delta \lambda, \Delta \phi) \to s(\Delta \lambda, \Delta \phi) - \Delta \lambda, \Delta \phi$

- Effective action: work with scale dependent parameters $\lambda \text{ and } \phi$

\[
\begin{aligned}
\int [D\Phi]_{0, V_0} &\quad \left( \phi(k_1) - \phi(k_2) \right) e^{-S_0[\Phi]} \\
\int [D\Phi]_{V_0, \lambda} &\quad \left( \phi(k_1) - \phi(k_2) \right) e^{-S_0[\Phi]} \\
&= \sum_{\lambda, V_0} \int [D\Phi]_{0, V_0} e^{-S[\Phi]}
\end{aligned}
\]

Effective action at scale $\lambda$

**Coarse grainy of geometries**