1. Define differential operators $P_{\mu}$, $\tilde{P}_{\mu}$, $Q_\alpha$, $\bar{Q}_{\dot{\alpha}}$, $D_\alpha$, $\bar{D}_{\dot{\alpha}}$ in superspace coordinates $\{x, \theta, \bar{\theta}\}$ as follows:

$$
P_{\mu} = i \partial_{\mu}, \quad \tilde{P}_{\mu} = -i \partial_{\mu},$$
$$Q_\alpha = -i \frac{\partial}{\partial \theta^{\alpha}} + \sigma^{\mu}_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_{\mu},$$
$$\tilde{Q}_{\dot{\alpha}} = -i \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + (\theta \sigma^{\mu}\epsilon)^{\dot{\alpha}} \partial_{\mu},$$
$$D_\alpha = -i \frac{\partial}{\partial \theta^{\alpha}} - \sigma^{\mu}_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_{\mu},$$
$$\bar{D}_{\dot{\alpha}} = -i \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - (\theta \sigma^{\mu}\epsilon)^{\dot{\alpha}} \partial_{\mu}.$$

Show that

\[
\{Q_\alpha, \tilde{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}, \quad \{D_\alpha, \bar{D}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} \tilde{P}_{\mu}, \quad \{Q_\alpha, D_\alpha\} = 0 = \{Q_\alpha, \bar{D}_{\dot{\alpha}}\}.
\]

2. Given a general scalar superfield $S(x, \theta, \bar{\theta}) = \phi(x) + \theta \psi(x) + \bar{\theta} \bar{\lambda}(x) + \theta^2 M(x) + \bar{\theta}^2 N(x) + \theta \theta^{\alpha} \sigma_{\alpha\beta} A_{\mu}(x) + \partial^2 \theta \bar{\lambda}(x) + \bar{\theta}^2 \bar{\lambda}(x) + \theta^2 \theta \rho(x) + \bar{\theta}^2 \bar{\rho}(x)$ find the supersymmetry transformations of the component fields. Remember that $\delta_{\xi} S = (i \xi Q + i \bar{\xi}\bar{Q}) S$.

3. Given a chiral superfield $\Phi$ find the component expansion of $\log(\Phi)$ and $\Phi^n$, where $n$ is a natural number.

4. For a general real superfield $V = V^\dagger$ find the D-component of $V^n$, where $n$ is a natural number.
5. Define a "spurion" superfield $Z = a + b\theta^2 + b^*\bar{\theta}^2 + c\theta^2\bar{\theta}^2$, where $a, b, c$ are complex numbers. Does $Z$ behave like a proper superfield under supersymmetry transformations? Find the component expansion for the Lagrangian density

$$\int d^4\theta Z \Phi^\dagger \Phi,$$

where $\Phi$ is a chiral superfield. Is the theory described by this Lagrangian density supersymmetric?

6. Find the component expansion of the Lagrangian density

$$\int d^4\theta \left( \Phi_1^\dagger e^{-V} \Phi_1 + \Phi_2^\dagger e^{+V} \Phi_2 + 2\xi V \right)$$

$$\int d^2\theta \left( m\Phi_1\Phi_2 + \frac{1}{M}(\Phi_1\Phi_2)^2 - \frac{1}{4}W^\alpha W_\alpha \right) + h.c.$$

Use the Wess-Zumino gauge for the vector superfield, $V = \theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta^2\lambda(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}D(x)$. The $\Phi_{1,2}$ are chiral superfields with charges $-1, +1$ respectively.

7. Show that one can define components of a chiral superfield $\Phi$ as follows:

$$\mathcal{A} = \Phi|_0,$$

$$\chi_\alpha = \frac{i}{\sqrt{2}}D_\alpha \Phi|_0,$$

$$\mathcal{F} = \frac{1}{4}D^2\Phi|_0,$$

where $|0$ means $\theta_\alpha = \bar{\theta}_\beta = 0$.

8. Consider an $SU(2), N = 1$ supersymmetric theory with three chiral superfields on the adjoint representation of $SU(2): \Phi_{1,2,3}$. The superpotential is given by

$$W = \epsilon_{ijk}Tr \Phi_i[\Phi_j, \Phi_k],$$

where $\epsilon_{123} = 1$. The Kähler potential is the minimal renormalizable one. Is supersymmetry broken in this model? Next, add to the superpotential

$$\delta W = \sum_i m_i Tr \Phi_i^2.$$

What about supersymmetry breaking in the presence of these terms? Show that the equations of motion become $[\Phi_i, \Phi_j] = \epsilon_{ijk}m_k\Phi_k$. Which matrices satisfy these equations?